Circuit Training – L'Hospital's Rule **Name** _____

Directions: Beginning in cell #1, evaluate the limit without the use of technology. If L'Hospital's Rule applies (and it will on the majority but not on all of these), indicate that it applies and then you may use it to arrive at your answer. Search for your answer, call that cell #2, and proceed in this way until you complete the circuit. You may need to include additional sheets of paper to show your best work.

Answer: 1 #__1__ $\lim_{x\to -3} \frac{x^2-9}{x+3}$	Answer: $\frac{5}{2}$ # _____ $\lim_{x\to\infty} \frac{5x^3}{e^{2x}}$
Answer: 5 # _____ Given: $f(3)=2,\ f'(3)=-6,$ $g(3)=0,$ and $g'(3)=12$ $\lim_{x\to 3} \frac{f(x)-2}{g(x)}$	Answer: $-\frac{3}{e}$ # _____ $\lim_{h\to 0} \frac{3^{h-2}-\frac{1}{9}}{h}$
Answer: $-\frac{1}{3}$ # _____ $\lim_{x\to 0} \frac{4\sin^2 x-4x}{3\sin 2x}$	Answer: -6 # _____ $\lim_{y\to \frac{5}{2}} \frac{2y-5}{4y^2-25}$
Answer: does not exist # _____ $\lim_{x\to\infty} \frac{2x-x^7}{3x^7-2x^3+4x+1}$	Answer: -5 # _____ $\lim_{x\to 0} \frac{\int_0^{x^2} \sin t dt - x^2}{\cos^2 x-1}$
Answer: 2 # _____ $\lim_{x\to 0} \frac{2e^x-2}{5x}$	Answer: $\frac{1}{10}$ # _____ $\lim_{x\to\infty} \frac{3x+2}{2x+3}$

Answer: $\frac{\ln 3}{9}$

_____

$$\lim_{x\to 3} \frac{(x-3)^2+\sin(\pi x)}{\frac{1}{x}-\frac{1}{3}}$$

Answer: $\frac{3}{2}$

_____

$$\lim_{a\to 1} \frac{a^2-1}{\ln a}$$

Answer: 6

_____

$$\lim_{x\to 0} \frac{3x^2+5x}{\ln(x+1)^2}$$

Answer: $-\frac{1}{2}$

_____

$$\lim_{\theta\to 0} \frac{\sin\theta}{2\theta}$$

Answer: $\frac{2}{5}$

_____

$$\lim_{x\to 3} \frac{x^2-36}{x+6}$$

Answer: $-\frac{2}{3}$

_____

$$\lim_{a\to 25} \frac{\sqrt{a}-5}{25-a}$$

Answer: 0

_____

$$\lim_{\alpha\to \frac{\pi}{2}} \frac{2\alpha}{\sin\left(\alpha-\frac{\pi}{2}\right)}$$

Answer: -3

_____

$$\lim_{h\to 0} \frac{\tan 5h}{h}$$

Answer: -1

_____

$$\lim_{x\to 5} \frac{\int_5^x t\,dt}{5-x}$$

Answer: 9π

_____ Given the following table of values:

t	$f(t)$	$f'(t)$	$g(t)$	$g'(t)$
2	7	12	7	10

$$\lim_{t\to 2} \frac{f(t)-g(t)}{\frac{t-2}{3}}$$

Answer: $\frac{1}{2}$

_____

$$\lim_{x\to e} \frac{3\ln(x)-3}{e-x}$$

Answer: $-\frac{1}{10}$

_____

$$\lim_{x\to 0} \frac{\tan^{-1}x}{x^2-x}$$

Circuit Training – Mixed Applications of the Derivative **Name**_____

Directions: Beginning in cell #1, use calculus to help answer the question. Show how the calculus finds or confirms your answer. *No technology* should be used at all for this circuit. To advance, search for your answer and that becomes #2. Continue in this manner until you complete the circuit. Attach additional sheets if the boxes do not provide sufficient space for you to show good work.

Answer: 17

__1__ Write the equation of the tangent line to the graph of $f(x) = \frac{1}{3}x^3 - x^2 + 7x - 2$ at $x = 3$.

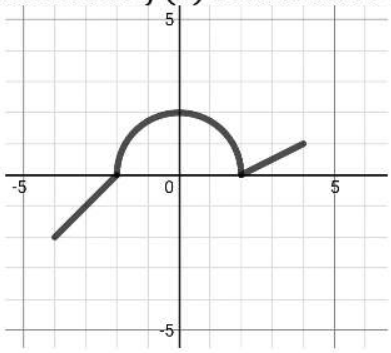
To advance in the circuit, use the tangent line to approximate $f(2.9)$.

Answer: -7

_____ A particle travels horizontally so that its position at any time, *t*, on the closed interval $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ is given by $x(t) = \frac{3t}{\pi} + \cos(t)$. Find the particle's position when its acceleration is -1.

Answer: -2

_____ The graph of two line segments and a semi-circle shows $f'(x)$ for $-4 \leq x \leq 4$. On what open interval is $f(x)$ both concave down and increasing? Explain.



To advance in the circuit, at which of the following *x*-values is $f(x)$ both concave down and increasing?

-1	1	3

Answer: 18

_____ The area of a square is expanding at a rate of 120 cm^2/min. Find the rate at which the side of the square is changing (in cm/min) when the area is 16 cm^2.

Answer: -9

_____ An advanced biology student wants to test the effects of a chemical fertilizer versus natural fertilizer, so she plans to divide a rectangular garden in half by running a fence parallel to two of the exterior sides. She has a fixed amount of fencing (36 m) and she wants to have fencing around the entire garden in addition to the fence down the middle. Find the maximum area of the natural fertilizer garden.

Answer: 2

_____ Where is the graph of $y = -x^4 + 50x^2 - 40$ both increasing and concave down?
Write your answer in interval notation: _____

To advance in the circuit, which of these three numbers is on the answer interval?

-7	1	7

Answer: 4

_____ Find the minimum value of the function $f(x) = xe^x$. Confirm that this is the absolute minimum. The minimum value is irrational, but the x-coordinate of the minimum is rational. To advance in the circuit, search for the x-coordinate of the minimum.

Answer: 7

_____ The function $f(x)$ has domain all reals and is differentiable everywhere. If the slope of the tangent line to f at $x = 3$ is the same as the average rate of change of $f(x)$ on the closed interval $[-7, 7]$, $f'(3) = -2$ and $f(-7) = 22$, find $f(7)$.

Answer: 15

_____ Find the maximum value of $g(x) = 12 - \ln(x^2 + 1)$ on the closed interval $[-1, 5]$.

Make sure to show that you check endpoints!

Answer: 1

_____ Evaluate the limit: $\lim_{\theta \to 0} \frac{\tan(5\theta)}{\ln(\theta+1)}$

Answer: 12

_____ A circle's circumference is shrinking at a rate of $\frac{3}{2}\pi$ cm/min. How fast is the area of the circle changing (in cm^2/min) when the circumference is 12 cm?

Answer: 16

_____ The function $g(x)$ is differentiable with selected values shown in the table. Estimate $g'\left(\frac{4}{5}\right)$.

x	0	$\frac{4}{5}$	$\frac{3}{2}$	2	$\frac{7}{3}$
$g(x)$	3	8	6	5	5

Answer: 5

_____ During a severe rainstorm water enters a catch basin at a rate of $R(t) = -t^2 + 4t$ (gallons/hour) where x is time in hours over the course of four hours $0 \leq t \leq 4$. Water leaves the catch basin via a drain at the rate of $L(t) = 2t$. Find the time, $t, 0 \leq t \leq 4$, at which the water in the catch basin is decreasing at the greatest rate.

Answer: 27

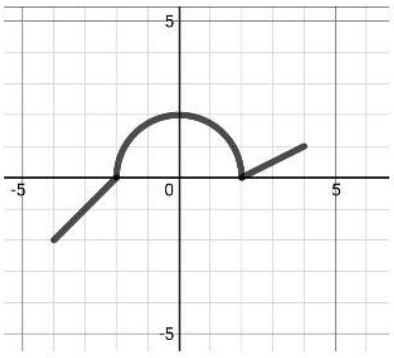
_____ Verify that $f(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem (MVT) on the closed interval [9, 25]. Then, find all numbers "c" that satisfy the conclusion of the MVT.

Answer: -1

_____ Given that $h(1) = 16$, and $h'(1) = 10$.
Use the linearization of $h(x)$ at $x = 1$ to estimate $h(1.1)$.

Answer: -6

_____ The graph of two line segments and a semi-circle shows $f'(x)$ for $-4 \leq x \lneq 4$. For what value of x on the closed interval $[-4, 4]$, does $f(x)$ have an absolute minimum? Justify.



Circuit Training – Applications of the Integral **Name** _____

Directions: Beginning in cell #1, do the work necessary to answer the question. Then, search for your answer. Mark that cell #2 and continue in this manner until you complete the circuit. Note: You should not need a calculator for any of these questions!

Answer: $\frac{\pi}{2}(e^2 - 3)$

__1__ Sketch the graph of $y = 3\sin x$, then shade the region under the curve from $x = 0$ to $x = \pi$.
Calculate the area of the shaded region.

Answer: 1

_____ A toy car's acceleration was measured at regular intervals over the course of a 12-second race. Using three sub-intervals of equal width, find the midpoint sum to estimate the velocity of the toy car at time $t = 12$ seconds.

t	0	2	4	6	8	10	12
$a(t)$	0	3	4	– 1	6	2	2

Answer: 10

_____ The function $f(x)$ is continuous on the closed interval $[0, 20]$ and has values shown in the table below. Use the four sub-intervals indicated in the table to find the trapezoidal approximation of $\int_0^{20} f(x)dx$.

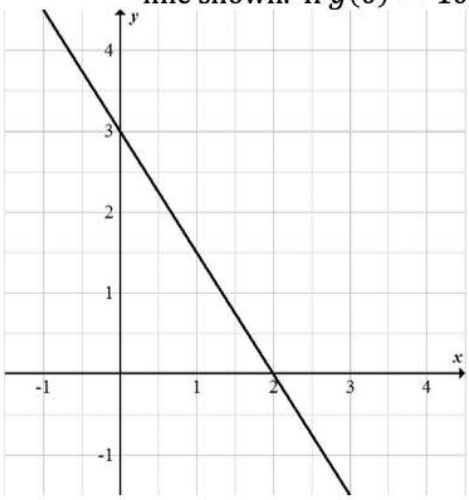
x	0	2	9	15	20
$f(x)$	– 3	0	3	5	– 4

Answer: $\frac{32}{27}$

_____ Find the particular solution $y = f(x)$ for the differential equation $\frac{dy}{dx} = \frac{x}{y}$ given $f(4) = -1$. To advance in the circuit, find $f(2\sqrt{10})$.

Answer: $\frac{13}{2}$

_____ The graph of g', the derivative of g, is the line shown. If $g(0) = 10$, then $g(2) = ?$



Answer: $\frac{1}{3}\left(16\sqrt{2} - 8\right)$

_____ Suppose $\int_2^x f(t)dt = 3x^2 + 2x + 1$. Find $f(-2)$.

Answer: 6

_____ What is the average value of the function $g(x) = x^3 + 5$ from $x = 0$ to $x = 2$?

Answer: 16

_____ Calculate the volume of the solid generated when the region bounded by $y = \sqrt{x - 1}$, the vertical line $x = 5$ and the x-axis is rotated about the x-axis.

Answer: 8π

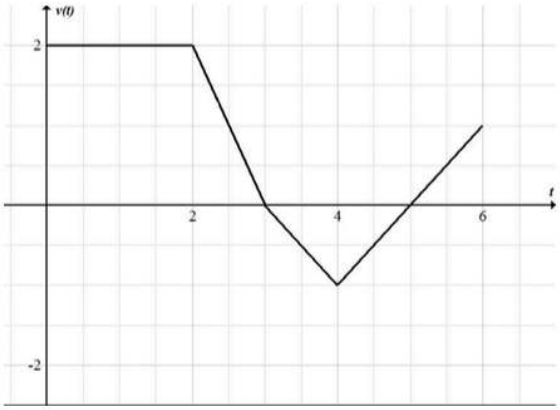
_____ Let $w(t) = \int_a^{t^2} \frac{1}{x^2-1} dx$. Find $w'\left(\frac{1}{2}\right)$.

Answer: 7

_____ Find the area of the region completely enclosed by the graphs of $y = 3x^2$ and $y = 2x + 1$.

Answer: 34

_____ A fly craws horizontally along a kitchen window sill. The graph shows the velocity (in cm/sec) of the fly. What is the total distance (in cm) traveled by the fly from $t = 0$ to $t = 6$ seconds?



Answer: $5(1 - e^{-48})$

_____ Sketch the region bounded by the lines y = 1 and x = 1, and by the function y = lnx. Calculate the volume of the solid generated when this region is rotated about the y-axis.

Answer: – 5 # _____ Water collects in a rain barrel during a storm at a rate of $R(t) = t\sqrt{t^2+4}$ cubic inches / hour. How much water (in cubic inches) is in the rain barrel at the end of two hours?	Answer: $-\frac{16}{15}$ # _____ The rate at which Texas oil fields harvest oil is modeled by the function $R(t) = 30e^{-6t}$ where t is in years and $R(t)$ is in billions of barrels /year. How much oil (in billions of barrels) will be harvested over the next eight years?
Answer: 13 # _____ Find the particular solution $y = g(x)$ for the differential equation $\frac{dy}{dx} = y\sec^2 x$ given $g(0) = e$. To advance in the circuit, find $g\left(\frac{3\pi}{4}\right)$.	Answer: – 10 # _____ Let $s(t)$ and $v(t)$ be position and velocity functions, respectively. If $v(t) = 3\sin t + \frac{6}{\pi}$ and $s(0) = -2$, find $s(\pi)$.